

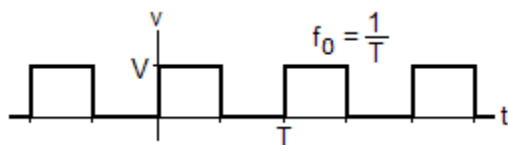
Experiment 44: Band Pass Filter / FFT / Square Wave

Introduction

According to Fourier analysis, a square wave may be considered to be a superposition of an infinite number of odd harmonic frequencies whose amplitudes decrease inversely with frequency. The fundamental is the lowest frequency of the square wave. The Fourier series of a square wave voltage, whose peak-to-peak amplitude is V , and average value is V_{AVE} , may be expressed as:

$$v(t) = V_{AVE} + \frac{2V}{\pi} \sum_n \frac{1}{n} \sin(2\pi n f_0 t), \text{ where } n \text{ is an odd}$$

integer and the fundamental frequency is f_0 .



If a square wave is applied to a band-pass filter, the wave's frequency components will be affected by the filter's frequency response. The filter's output frequency spectrum will be different than the input frequency spectrum.

In Fourier analysis, the concepts of "time domain" and "frequency domain" are important to understand. The square wave input to the filter and output from the filter are considered to be expressed in the time domain when its amplitude is expressed as a function of time. An oscilloscope normally plots waveforms in the time domain. When the amplitude of the waveform is expressed as a function of frequency, it is expressed in the frequency domain.

Using Fourier analysis, it is possible to convert time domain expressions into the frequency domain, and frequency domain expressions into the time domain. This may be done by a computer algorithm called the "Fast Fourier Transform", or "FFT", which is built into many analysis software packages, including *PSpice* and *LTspice*. The objective of this experiment is to investigate the relationship between time domain analysis and frequency domain analysis. Acquired time domain data will be compared to simulation data using *OrCAD PSpice*.

Although the Fourier series of any time domain waveform consists of an infinite number of harmonic frequencies, an adequate approximation may be obtained using only the first several terms of the series. This exercise will involve the average value of the series plus the first 3 harmonics. These are:

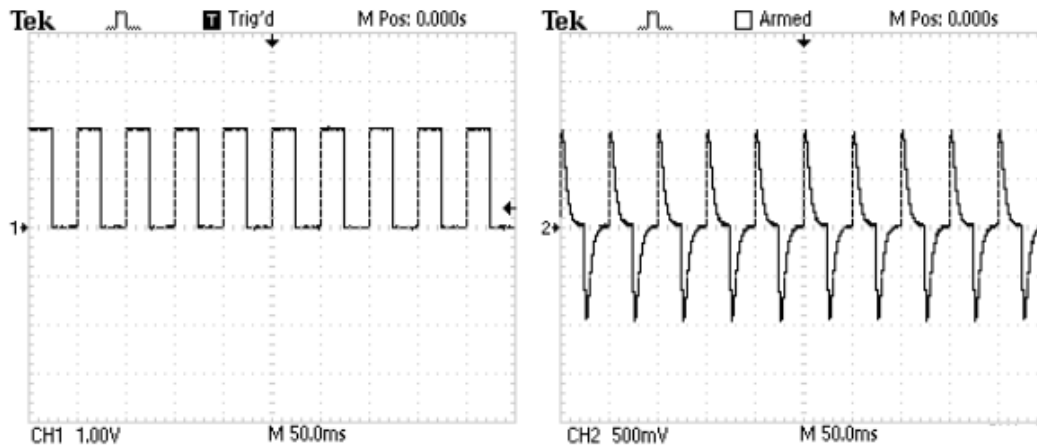
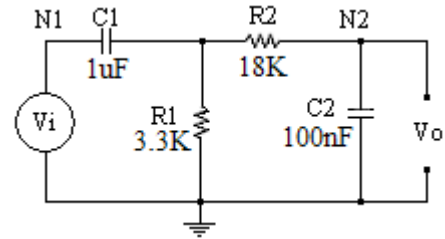
$$v(t) = V_{AVE} + \frac{2V}{\pi} \sin(2\pi f_0 t) + \frac{2V}{3\pi} \sin(6\pi f_0 t) + \frac{2V}{5\pi} \sin(10\pi f_0 t)$$

Procedure

Equipment and Parts

Function Generator, Digital Oscilloscope with FFT, and Breadboard
Resistors: 3.3K, 18K, ¼ watt, 5%
Capacitors: 1µF (non-polarized), 100nF, 5%.

1. Connect the circuit on the right. Connect the oscilloscope channel 1 to node N1 and Channel 2 to node N2.
2. V_i is the function generator set to produce a 20Hz, 2V peak-to-peak, square wave with a 1 volt offset (goes between 0 and 2V).
3. Display 10 cycles of the input and output waveforms as shown below.



It may be helpful to read the section on displaying the Fourier spectrum of your instrument's user manual.

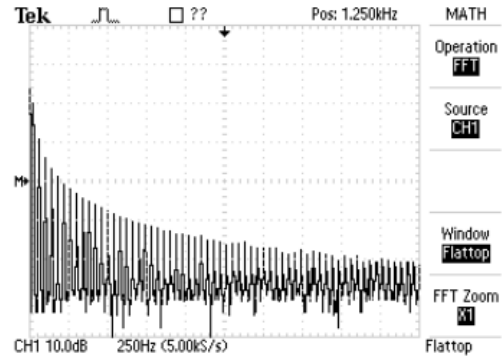
The TDS1002 uses a "Math FFT" algorithm to generate the Fourier spectrum from the acquired time domain data. The time domain waveform needs to be set up carefully to get an accurate display of the spectrum. Please note the following:

- a. The FFT spectrum is calculated from the center 2048 points of the time domain waveform. Since there are 2500 points in the 10 divisions of the entire display, the FFT is calculated from about the center 8 divisions.
- b. There is a tradeoff between frequency resolution and the bandwidth of the displayed spectrum (due to aliasing).
- c. The highest frequency (and bandwidth) that can be measured accurately by a digitizing oscilloscope is one half the sample rate (the Nyquist frequency).
- d. The TDS1002 transforms 2048 time domain points to 1024 frequency domain points resulting in a spectrum whose bandwidth is equal to the Nyquist frequency.
- e. The amplitude is displayed in dB where 0 dB is equal to 1 V rms.

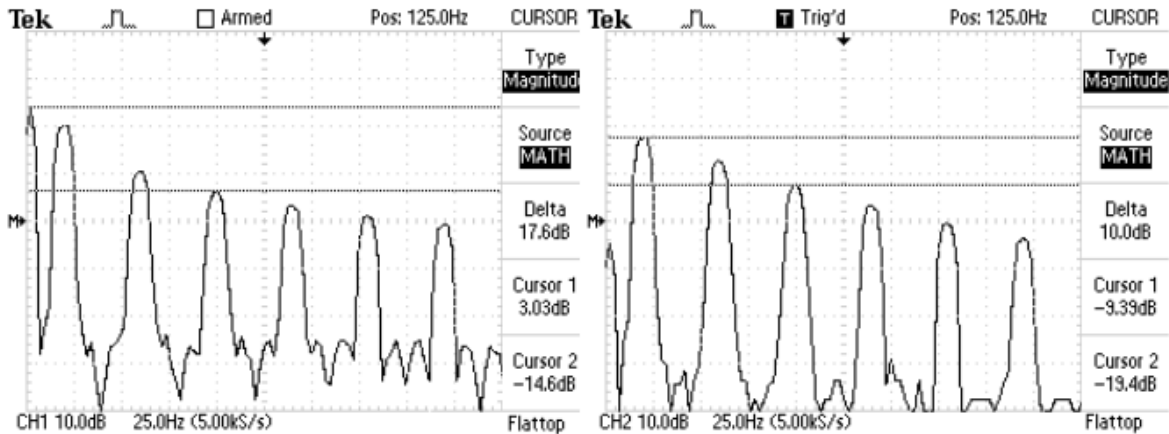
$$\text{dB}_{\text{VRMS}} = 20 \log \frac{V_{\text{RMS}}}{1\text{V}_{\text{rms}}} .$$
- f. Use the cursors to determine the amplitudes of the harmonics.
- g. Use data capture such as the TekXL toolbar for greater accuracy

- To obtain the FFT spectrum of channel 1: Push the "MATH MENU" button, select "FFT", set the MATH FFT source to channel 1.

Note the resulting display on the right. The frequency scale is 250Hz/Div. The amplitude scale is in dB, where 0dB = 1V rms.



- The frequency scale needs to be expanded by a factor of 10 in order to accurately measure the frequency components. This could be done with the time base (sec/div) control. However, this would reduce the bandwidth from 2500Hz to 250Hz. The TDS1002 has an FFT zoom control that provides a zoom up to 10. A zoom factor of 10 was used to obtain the input (CH1) and output (CH2) frequency spectrums shown below.



Cursors should be used to measure the amplitudes. The input frequency spectrum (channel 1) above shows an rms amplitude value of 3dB at 0Hz. This corresponds to 1.414V rms, which is equal to a 1V average value (DC offset). The fifth harmonic at 100Hz shows an amplitude of -14.6dB, corresponding to a peak voltage of 0.254V. The output frequency spectrum shows the fifth harmonic amplitude to be -19.4dB.

- Display the first six harmonics of the input and output spectra of your band pass filter. Record the amplitude of the DC component and the first three harmonics of the input and output spectra (DC, 20Hz, 60Hz, 100Hz).

V_{iDC} : _____ dBV V_{i20} : _____ dBV V_{i60} : _____ dBV V_{i100} : _____ dBV

V_{oDC} : _____ dBV V_{o20} : _____ dBV V_{o60} : _____ dBV V_{o100} : _____ dBV

Analysis

Note: Use a spreadsheet as applicable to do the analysis below.

1. The Fourier series of the input waveform is that of a 2V peak-to-peak amplitude square wave with a 1.0V DC component (average value). Determine the theoretical magnitude of the waveform's DC component, fundamental, third harmonic and fifth harmonic. Convert your calculated results to dBV using the following equation:

$$\text{dBV} = 20 \log \frac{V_{\text{rms}}}{1V_{\text{rms}}}$$

Compare your measured values to the theoretical values by calculating the percent difference between them.

2. Simulate the filter circuit and use the FFT feature to obtain the frequency spectrum of the output. Compare the simulated results to your calculated results from step 1 above.
3. Simulate the circuit and use the frequency sweep analysis to obtain a Bode Plot of the filter's response (similar to plot shown below). Compare the results to analysis steps 1 and 2 above.

