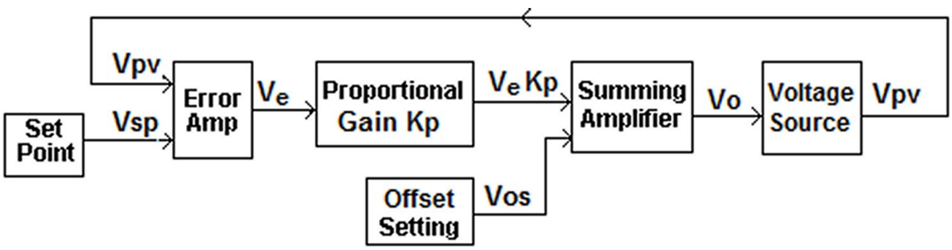
Chapter 4: Proportional Mode Control System

The amount of corrective action applied by a proportional mode controller to a process is proportional to how far the process variable is from the set point. This allows the control system to reach the set point faster and reduces the possibility of overshoot and undershoot.

An example of a proportional mode control system is presented in the block diagram below. This is basically a voltage source whose output voltage can be varied by the set point setting.



The output voltage, **Vpv**, of the voltage source is the process variable. **Vpv** is compared to the set point voltage, **Vsp** by a differential amplifier with a voltage gain of one. The difference is the error voltage which can be expressed as: **Ve = Vsp – Vpv.**

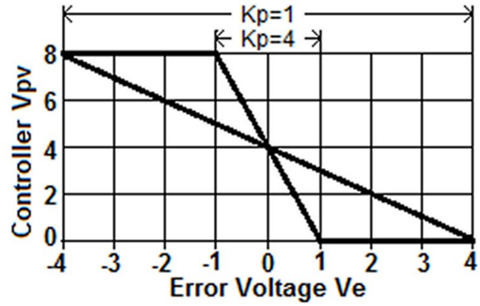
Ve is amplified by an operational amplifier with a gain of Kp. This amplified error voltage controls the voltage source output voltage, **Vpv**, in such a way as to reduce the error voltage (negative feedback). The output voltage of the controller, **Vo**, can be expressed as: **Vo = Ve·**Kp **+ Vos.**

Study the block diagram above. When the error voltage is zero, the controller output voltage, **Vo**, will be the offset voltage, **Vos**. Typically **Vos** is set so that **Vpv = Vsp** at a desired resting or equilibrium value. For this one setting of **Vsp** the error voltage will be zero.

When **Vsp** is changed, the voltage needed to drive **Vpv** to the new setting is the amplified error voltage, **Ve·**Kp. Increasing the value of Kp will reduce the amount of error voltage needed so that the new setting will be approached with less error.

Proportional Band

The range of error voltage which produces an output that is proportional to the error is called the "proportional band".

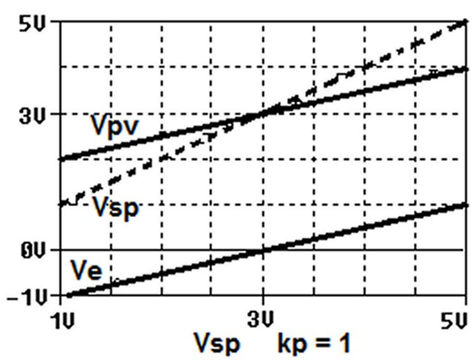


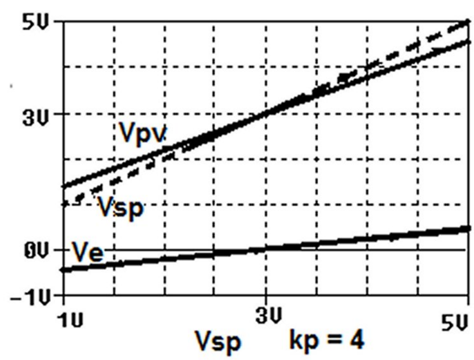
The graph on the right is for a system whose process variable, **Vpv**, can vary from 0 to 8 volts.

When **Vsp** = 4 volts, **Vpv** = 4 volts and **Ve** = 0.
When Kp = 1, the proportional band is ±4 volts.
When Kp = 4, the proportional band is ±1 volt.

When the error voltage is outside of the proportional band, the process variable will be the output saturation voltage, either 0 volts or 8 volts.

The graphs below show the control systems response for the voltages **Ve** and **Vpv** as the set point voltage, **Vsp**, is varied from 1.0 volts to 5.0 volts for proportional gains of one and four. Equilibrium value of **Vpv** is set to 3.0 volts. Ideally, **Vpv** should coincide with the dashed line.





Steady State Response

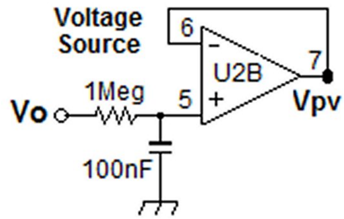
Given Vo = Ve·Kp + Vos and that the steady state value of Vpv = Vo, the steady state value of Vpv is calculated below:

$$\mathbf{Vpv} = \mathbf{Vo} = \text{Kp}(\mathbf{Vsp} - \mathbf{Vpv}) + \mathbf{Vos} \quad \Rightarrow \quad \mathbf{Vpv} = \left(\frac{\text{Kp}}{1+\text{Kp}}\right)\mathbf{Vsp} + \left(\frac{1}{1+\text{Kp}}\right)\mathbf{Vos}$$

The result shows that as Kp approaches infinity, Vpv approaches Vsp.

Transient Response

The transient response of a control system occurs from the time a new set point is initiated to the time the system reaches and settles on the new set point value.



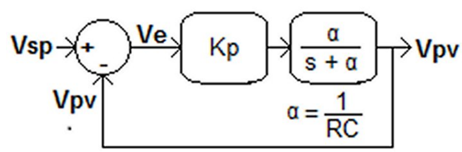
This exercise uses the voltage source shown on the right for the controlled process. An RC network with a 0.1 second time constant introduces (simulates) a time delay to the process.

Time or frequency dependant transfer functions must be used to calculate the transient response of the control system. In this case, only the time or frequency dependence of the voltage source needs to be considered because it has a much slower time response than the rest of the system. That is, the rise and fall times of the other system components can be neglected.

The frequency response of the control system can be easily calculated as the frequency response of the voltage source (given, in this case, that the cutoff frequency of this voltage source is much lower than the cutoff frequency of the other system components). However, the transient response calculation is not as easy because the system's response to a step change in the set point needs to be determined.

Algebra alone is not sufficient for this calculation. If you are not familiar with the math, you may skip down to the final result.

The control system's transient response is calculated below using the Laplace transform method. A simplified block diagram is shown on the right.



$$\mathbf{Vpv} = \mathrm{Kp}(\mathbf{Vsp} - \mathbf{Vpv})\frac{\alpha}{s+\alpha} \quad \Rightarrow \quad \mathbf{Vpv}\left(1 + \frac{\alpha}{s+\alpha}\mathrm{Kp}\right) = \frac{\alpha}{s+\alpha}\mathrm{Kp}\mathbf{Vsp}$$

$$\mathbf{Vpv} = \frac{\frac{\alpha}{s+\alpha}\mathrm{Kp}\mathbf{Vsp}}{\left(1 + \frac{\alpha}{s+\alpha}\mathrm{Kp}\right)} = \frac{\alpha \mathrm{Kp}}{s + \alpha(1+\mathrm{Kp})}\mathbf{Vsp}$$

$$\mathbf{Vsp} = \frac{1}{s}, \text{ a unit step, then, } \quad \mathbf{Vpv} = \frac{\alpha \mathrm{Kp}}{s(s + \alpha(1+\mathrm{Kp}))}$$

Expanding result by partial fractions and transforming to the time domain yields:

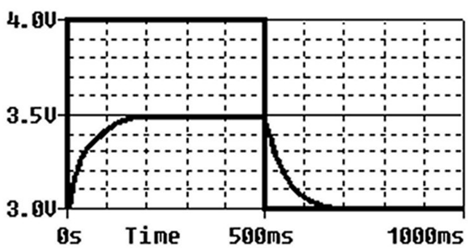
$$\mathbf{Vpv} = \left(\frac{\text{Kp}}{1+\text{Kp}}\right)\left(1-e^{-\alpha(1+\text{Kp})t}\right)$$

This result is for a unit step with no offset.

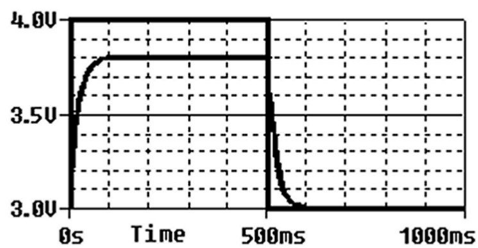
Final result (A is the step size in volts, Vos is the offset voltage, and t is time in seconds):

$$\mathbf{Vpv} = \left(\frac{\text{AKp}}{1+\text{Kp}}\right)\left(1-e^{-\alpha(1+\text{Kp})t}\right)+\mathbf{Vos.} \quad \mathbf{At\ (t=\infty),\ Vpv} = \left(\frac{\text{AKp}}{1+\text{Kp}}\right)+\mathbf{Vos.}$$

This result shows that as the value of Kp increases the step size approaches A and the response time (rise and fall times) decreases. This is shown by the simulation results below.



A = 1 Kp = 1

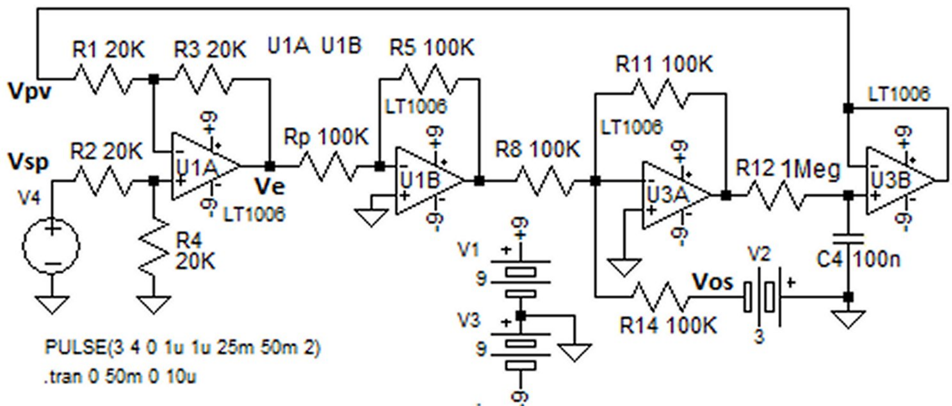


A = 1 Kp = 4

Simulation results above show the control system's transient response in going from the equilibrium set point of 3.0 volts to a set point of 4.0 volts and back to 3.0 volts. A Kp of 1.0 results in a steady state error of 0.5 volts when going to 4.0 volts. A Kp of 4 results in a steady state error of 0.2 volts when going to 4.0 volts and a much shorter rise times, fall times, and settling times.

Simulation of Proportional Control

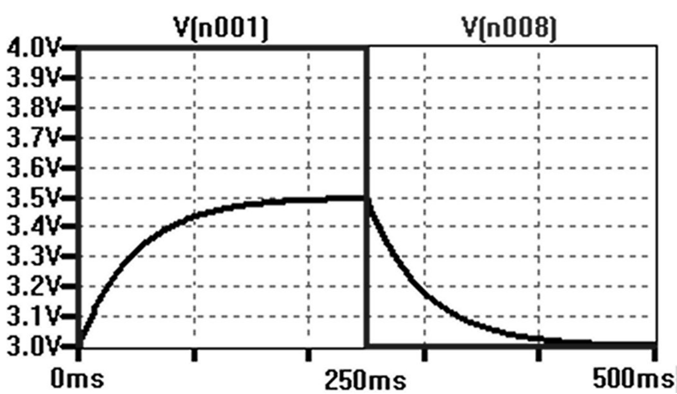
Study the LTspice circuit below. U1A is a unity gain differential amplifier Its output is the error voltage, **Vsp – Vpv**.



U1B sets the proportional gain: Kp = R5/RP. U3A sums the proportional output, **Ve**Kp and the offset voltage, **Vos.** R12 and C4 add a 100mS time constant to the process.

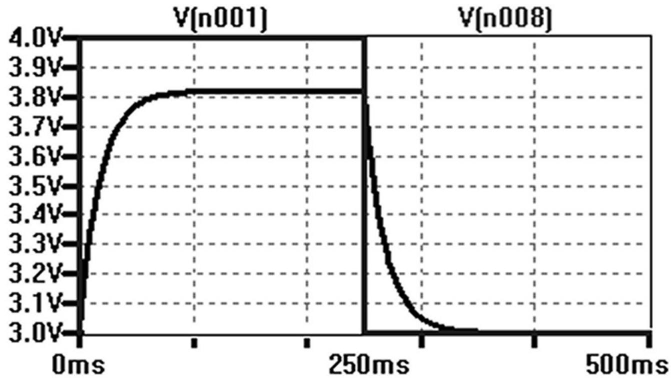
Set the simulation to "Transient' with a stop time of 50mS. Plot the voltages **Vsp** and **Vpv**.

Study the graph on below. The response for the first 250mS shows the capacitor charging from 0.0 volts to 3.5 volts. During this time the error voltage changes from 4.0 volts to 0.5 volts.



The next 250mS interval show that the error voltage settles on 0.0 volts when the set point voltage is the equilibrium value of 3.0 volts.

Experiment with the simulation by changing the proportional gain, Kp, the set point, **Vsp**, and the offset voltage, **Vos**. The result below is for Kp = 4.



Compare the simulation results to the results predicted by the equation:

$$\mathbf{Vpv} = \left(\frac{\mathrm{AKp}}{1+\mathrm{Kp}}\right)\left(1-e^{-\alpha(1+\mathrm{Kp})t}\right)+\mathbf{Vos.} \qquad \mathrm{Kp}=\frac{100\mathrm{k}}{\mathrm{Rp}}$$

Experiment 6: Proportional Mode Control System

The PID-X1controller board may be used in this exercise to implement a proportional control system. The controlled process is a voltage source whose set point voltage is switched between 3 volts and 4 volts by a square wave. The transient and steady state response of a proportional control system to a change of set point will be measured for several values of amplifier gain, Kp. The responses will be compared to simulations and calculations. This lab experiment demonstrates the control system concepts of proportional band, proportional error, response time, and settling time.

Parts and Equipment Required

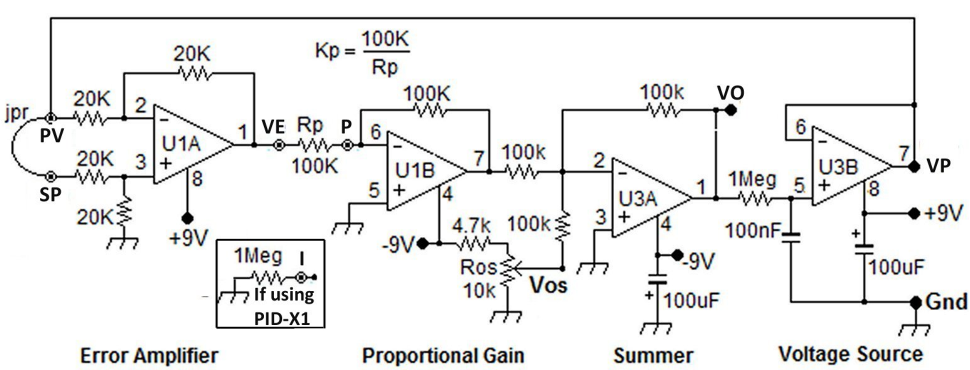
Oscilloscope, DMM, Function Generator.
Power Supply: ±9 to ±12 volts.
Controller board PID-X1 (or build the circuit on a breadboard).
Resistors: 10k, 100k, 1Meg, all ¼W, 5%.

Procedure Part 1: Steady State Response

1. The control circuit schematic diagram is given below. If using the PID-X1 control board, make the connections as shown below. In addition, connect a 1 megohm resistor from pin I (integrator input) to ground. Refer to the picture on the next page.

   If you are not using the PID-X1 control board, layout and build the circuit as carefully as possible. Keep the number of wires to the absolute minimum. Observe the numbers on the ICs. For example, U1A and U1B are in the same package.

   Observe the polarities on the 100μF electrolytic capacitors.

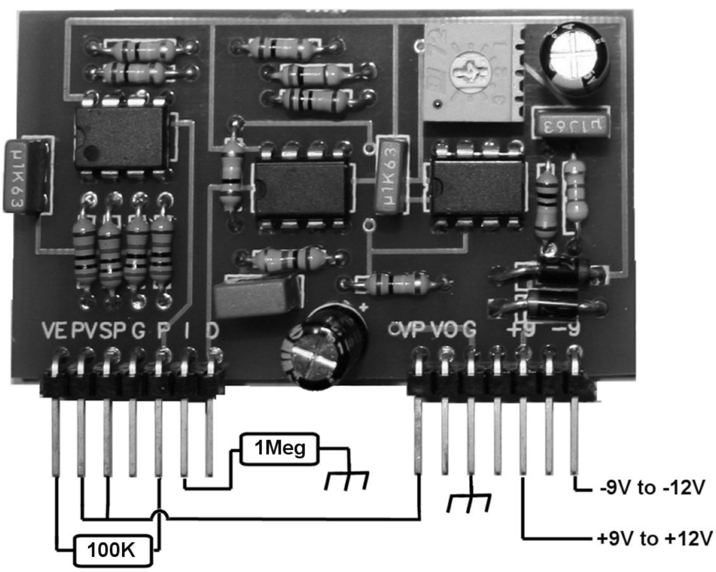


PID-X1 note: **Vpv** = **PV**, **Vsp** = **SP**, **Ve** = **VE**, **Vos** = **VO** when **Ve** = 0.

Layout the circuit in the same order as the schematic, U1A error amplifier circuit on the far left and the transistor on the far right. It is important that you can easily identify each functional block of the circuit on the breadboard.

A jumper wire “jpr” is connected between **PV** and **SP** temporarily to set the equilibrium value of **VP** (the zero error value of **VP**).

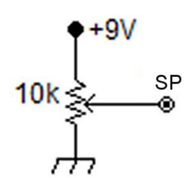
1b. If you are using the PID-X1, plug it into your breadboard and make the connections shown below.



2. Apply power to the circuit. Check that the voltage **VE** is zero. It should be very close to zero since **PV** = **SP** (due to the jumper wire).

   Adjust the potentiometer, Ros, to set **VO** to exactly 3.0 volts. Use a DMM to make the measurement. Measure the resulting value of **VP** and check that it is equal to **VO** (3V).

3. Turn off power. Remove the jumper between **PV** and **SP**. A variable voltage source needs to be connected to **SP**. This may be a separate power supply or use a potentiometer connected to +9V (or up to +12V) and ground as shown on the right.



4. Set **SP** to 3.0 volts. Verify that **VE** is close to 0 (less than 0.1 volts). Set **SP** to the following voltages and each time measure **VE** and **PV**: 0, 2, 3, 4, 5, 6, 7, 8 Record the measurements in the table below.

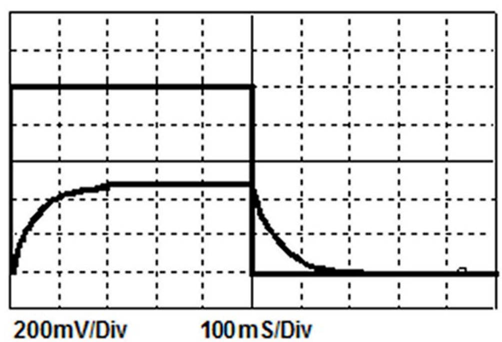
SP(Vsp)	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
PV(Vpv)									
VE(Ve)									

5. Remove the potentiometer from **SP**.

Procedure Part 2: Transient Response

1. Set the function generator to produce a 1 volt peak-to-peak, 1Hz, square wave with a 3.5 volt offset (so it goes between 3.0 volts and 4.0 volts). Connect the function generator to **SP**. Turn on power to control circuit.

Connect the oscilloscope channel 1 to **SP** and Channel 2 to **PV**. Set vertical inputs to DC coupling and 200mV/Div. Set horizontal to 100mS/Div and Trigger on channel 1. Set vertical position to obtain a display similar to that shown on the right.



3. From your oscilloscope display, determine the steady state value, rise time, and settling time of **PV** (**Vpv**). Record below.

Steady State ____________volts Rise Time ____________ mS

Settling Time ____________mS

4. Replace Rp in the controller circuit with a 10k resistor to set Kp to 10. Repeat steps 1 through 3 and record results below. You should reduce the oscilloscopes time per division to measure the rise time of **PV** more accurately.

Steady State ____________volts Rise Time ____________ mS

Settling Time ____________mS

Analysis Part 1

1. Calculate the expected steady state errors for Kp =1, Vsp = 2 volts, **SP** = 4 volts, and **SP** = 6 volts (should have zero error for **SP** = 3 volts). Compare calculations to measured results. Express the percent difference between the measurements and calculations.

2. Calculate the expected steady state errors for Kp =10, Vsp = 2 volts, **SP** = 4 volts, and **SP** = 6 volts (should have zero error for **SP** = 3 volts). Compare calculations to measured results. Express the percent difference between the measurements and calculations.

3. Calculate the controller's proportional band for the set point of 3.0 volts with Kp = 1 and Kp = 10. Assume that the op-amp saturation voltage is 1 volts less than the supply voltage and that the diode saturation voltage is -0.7 volts.

Analysis Part 2

1. Calculate the time constant of the RC filter in the voltage source (100nF capacitor and 1Meg resistor). Calculate α (this is also called the "decay rate" and has units of "nepers per second"). Compare the rise time of **PV** of the control system with Kp = 1 to the calculated time constant.

2. Compare the measured results of procedure part 2, steps 3 and 4, to calculated results. Use the equation below with the appropriate values of Kp, **Vos**, and α (**Vos** = **VO** when **VE** = 0).

$$\mathbf{Vpv} = \left(\frac{AKp}{1+Kp}\right)\left(1 - e^{-\alpha(1+Kp)t}\right) + \mathbf{Vos.}$$

3. Calculate the effect of connecting a 220k ohm resistor in parallel with the 100nF capacitor. What happens to α? What happens to **PV** when **Vos** is 3.0 volts and **SP**, the set point. is 3.0 volts. What happens to **PV** when **Vos** is 3.0 volts and the set point is 4.0 volts.